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## Study of Knowledge-Based System (KBS) and Decision Making Methodologies in Materials Selection for Lightweight Aircraft Metallic Structures

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#### Abstract

This paper presents an overview of knowledge-based system (KBS) in the context of decision making methodologies used in materials selection for the design of light weight aircraft metallic structures. Overall aircraft weight reduction means substantially less fuel consumption and better efficiency. Part of the solution to this problem is to find a way to reduce overall weight of metallic structures in the aircraft. Two distinct multiple criteria decision making (MCDM) methodologies are presented with examples featuring a set of short-listed materials suitable in the design of the structures. Pre-defined constraint values, mainly mechanical properties, are employed as relevant attributes satisfying the design requirements. Presently, aluminum alloys with high strength-to-weight ratio have been second-to-none in most of the lightweight aircraft parts manufacturing. Magnesium alloys that are much lighter in weight and have impressive strength-to-weight ratios as alternatives to the use of aluminum alloys in the structures are examined using the methodologies. Ashby's approach of materials selection is generalized and materials are ranked based on the individual material index values. Finally, Materials are ranked based on the results obtained using the methodologies and are compared with those obtained using generalized Ashby's approach of materials selection. Any disparity among the individual materials ranking results are discussed.

**KEYWORDS:** Knowledge-based system, Materials selection, MCDM, AHP, TOPSIS, Ashby's charts

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**RESEARCH ARTICLE** 

# Study of Knowledge-Based System (KBS) and Decision Making Methodologies in Materials Selection for Lightweight Aircraft Metallic Structures

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## ABSTRACT

This paper presents an overview of knowledge-based system (KBS) in the context of decision making methodologies used in materials selection for the design of light weight aircraft metallic structures. Overall aircraft weight reduction means substantially less fuel consumption and better efficiency. Part of the solution to this problem is to find a way to reduce overall weight of metallic structures in the aircraft. Two distinct multiple criteria decision making (MCDM) methodologies are presented with examples featuring a set of short-listed materials suitable in the design of the structures. Pre-defined constraint values, mainly mechanical properties, are employed as relevant attributes satisfying the design requirements. Presently, aluminum alloys with high strength-to-weight ratio have been secondto-none in most of the lightweight aircraft parts manufacturing. Magnesium allovs that are much lighter in weight and have impressive strength-to-weight ratios as alternatives to the use of aluminum alloys in the structures are examined using the methodologies. Ashby's approach of materials selection is generalized and materials are ranked based on the individual material index values. Finally, Materials are ranked based on the results obtained using the methodologies and are compared with those obtained using generalized Ashby's approach of materials selection. Any disparity among the individual materials ranking results are discussed.

### **Keywords**

Knowledge-based system, Materials selection, MCDM, AHP, TOPSIS, Ashby's charts

### **INTRODUCTION**

Materials selection processes have been the most important aspects in product design and development. It is estimated that there are around 100,000 engineering materials [1] belonging to various families and classes of materials. This number is never decreasing, rather increasing with invention of new materials. Unlike in the early industrial era when materials used to be selected using trial and error approach, materials selection in engineering design has come a long way today. With advancement in technology and computer aided design tools with built-in data-base containing incredible amount of materials information, materials selection has become much more sophisticated. Many of the old engineering structures if built today would have a much lighter weight and yet be stronger. Similar is also true in the context of materials used in old aircrafts. Aircrafts designed in the last decade or two are much lighter in weight and more efficient. Invention of new materials alone that are lighter and stronger cannot solve the ever-existing problems of selecting an optimum material for an engineering design. It is crucial to know enough about a material in terms of how it performs in a design. Equally importantly, a right material selection algorithm and methodology is needed to select the best material for an engineering design for an optimum performance.

Materials selection in engineering design is solely governed by material properties. Information in engineering materials could primarily be divided into two main categories: data and knowledge. Data is defined as the results of measurements of properties, whereas knowledge represents the connection between items of the data [2]. Data of materials and what each data say about the materials together is called knowledge-based system (KBS). KBS is one of the most important tools in materials selection process in engineering design, without a complete understanding of which, it is impossible even to think of a product design. In the recent years KBS is readily available in various material databases and design software such as materials selection tools developed by GRANTA, a materials intelligence company. American Society of Metals handbook (ASM handbook) is another source of material data and its information.

Several multi-criteria decision making (MCDM) methodologies have been developed and proposed by engineers and researchers. Saaty [3] developed analytical hierarchy process (AHP) which is widely used in materials selection and decision making using pairwise comparison. The process in this methodology is quite simple and effective but lengthy. When the number of alternatives as well as the relevant attributes considered for the design increase, this method becomes increasingly complicated. Hwang and Yoon [4] developed a technique for order of preference by similarity to ideal solution (TOPSIS) to solve decision making problems. This method is fairly simple and measures relative closeness of alternatives to the positive and negative ideal solution. Ashby [5] [6] has made significant contribution in materials selection. It is seldom the case that performance of a component depends on just one attribute. It is almost always a combination of attributes that matter [5]. This gives an idea of plotting one attribute against the other in a chart for a range of materials. Ashby created such charts called Ashby's charts after his name. These charts include a range of materials in the material universe and contain a large body of information and correlate one attribute to the other for any material of interest. The first ever decision and optimization methodology was developed by S. Opricovic called VIseKriterijumska Optimizacija kompromisno Resenje (VIKOR). This method is based on a compromise solution as a feasible solution to a decision making problem, which is closest to the ideal solution, and

a compromise means an agreement established by mutual concessions [7]. Shanian and Savadogo [8] presented a material selection model using a multiple attribute decision making methodology called ELECTRE. This model uses the concept of outranking relationship, and the procedure is very lengthy. Rao [7] proposed improved compromise ranking method introducing AHP in VIKOR and considers materials selection attributes for the design application with their relative importance.

Even though a significant amount of research work has been done in the past towards decision making in materials selection, there is still a need of simpler methodology, precisely considering KBS that can accommodate any number of short-listed materials and relevant attributes. In this study, AHP and TOPSIS are discussed and applied to a set of short-listed materials and relevant attributes for materials selection in the design of lightweight aircraft metallic structures. As a simpler approach in materials selection, relevant materials indices could be identified using Ashby's method and materials could be ranked based on individual index values. This approach is considered in this paper and results are compared and validated with those obtained from AHP and TOPSIS.

#### LITERATURE REVIEW

Design engineers and decision makers use various methodologies available to decide which material to choose from among a number of alternatives. Analytical Hierarchy Process (AHP) is used to make a pairwise comparison among alternative materials as well as attributes in decision making. Selection of materials is always governed by its attributes and manufacturing processes [1]. There are two different approaches to materials selection. One is the material-first approach in which the design engineer selects materials based on material class and narrows it down to a selective set of materials with respect to their attributes satisfying the design requirements. The other is the process-first approach. In the latter approach, the design engineer selects materials based on the manufacturing process of materials. At the end, regardless of the type of approach, the materials selection process would end at the same conclusion. This paper considers material-first approach and materials have been short-listed based on their attributes rather than their processing governance.

#### **Analytical Hierarchy Process (AHP)**

AHP leads a design team through the calculation of weighing factors for decision criteria for one level of the hierarchy at a time. AHP also defines a pairwise comparison-based method for determining relative ratings for the degree to which each of a set of options fulfills each of the criteria [9]. AHP's application to the engineering design selection task requires that the decision maker first create a hierarchy of the selection criteria. This process starts with creating a matrix of size MxM where M is the number of attributes or the alternatives depending on what is being compared. The size of this matrix increases with the increase in the number of attributes as well as the alternatives. Each element in the matrix is denoted by r<sub>ii</sub>, which means that attribute i is compared with attribute j. An attribute compared to itself is always 1. That is if  $r_{ii} = 1$  when i=j and  $r_{ii} = 1/r_{ii}$ . For example, if the relative importance of attribute i to j is p, then the relative importance of attribute j to i is its reciprocal, 1/ p. The overview of certain matrix A of size MxM, where M is the number of attributes or the alternatives, is given in Equation 1 [7].

In this matrix, values of all the diagonal elements are 1 and the rest of the elements are either  $r_{ij}$  or  $1/r_{ij}$ . Table 1 presents the relative importance scale used in AHP. If the number of attributes are large, values in between can also be assigned. This definition of degree of importance varies from

Attributes
 1
 2
 3
 ...
 M

 1
 
$$a_{11}$$
 $a_{12}$ 
 $a_{13}$ 
 ...
 M

 2
  $a_{21}$ 
 $a_{22}$ 
 $a_{23}$ 
 ...
  $a_{1M}$ 

 A
 3
  $a_{21}$ 
 $a_{22}$ 
 $a_{23}$ 
 ...
  $a_{2M}$ 

 A
 3
  $a_{31}$ 
 $a_{32}$ 
 $a_{33}$ 
 ...
  $a_{3M}$ 

 ...
 ...
 ...
 ...
 ...
 ...
 ...

 M
  $a_{M1}$ 
 $a_{M2}$ 
 $a_{M3}$ 
 ...
 ...
  $a_{MM}$ 

#### Equation 1

one literature to another. Some of researchers have considered decimal values from 0.115 to 0.895 and numbers in between with equal intervals.

*Table 1: relative importance of material selection factors (a 5-point scale)* 

Degree of Relative Importance (r <sub>ij</sub> )	Definitions
1	Equally important
3	Moderately more important
5	Strongly more important
7	Very strongly important
9	Extremely important

The following steps are taken to complete the AHP process:

Step-1: A criteria comparison matrix [C] is created using relative importance ratings from Table 1.

Step-2: Matrix [C] is normalized by dividing each element in the matrix by sum of each column. This gives a new normalized matrix [Norm C].

Step-3: Each row of [Norm C] is averaged. This gives criteria weight vector {W}.

Step-4: A consistency check on comparison matrix [C] is performed by calculating the Consistency Ratio (CR). CR checks the consistency of the comparison matrix values assigned by the decision maker. If this value is less than 10 percent or 0.1, the criteria comparison matrix [C] is considered to be consistent and criteria weight {W} is valid. Otherwise, the decision maker has to go back to [C] and adjust the values.

Additional steps to perform the consistency check by calculating CR are given as follows [1]:

- a. Ealculate the weighted sum vector,  $\{W_s\} = [C] \ge \{W\}$ .
- b. Ealculate the consistency vector,  $\{Cons\} = \{W_s\} / \{W\}.$
- c. Gstimate Eigen value  $\lambda$  of the unit matrix given by [C]. This is the average value of {Cons}. In matrix theory, the Eigen values are a set of scaler quantities associated with a linear system of a matrix equation also known as characteristic roots. For any nth order polynomial, there are n number of characteristic roots. The largest of these roots is called the maximum Eigen value of the matrix and is represented with  $\lambda_{max}$ . In AHP, this value is the average of consistency vector {Cons}.
- d. Gvaluate the consistency index (CI) value. Equation 2 is used to calculate the CI value.

$$CI = \frac{(\lambda - n)}{(n - 1)} ;$$

Equation 2

Where n is the number of attributes or alternatives used in the pairwise comparison.

- e. F etermine the Random Index (RI) value. The RI values are the consistency index values for randomly generated versions of [C]. These values for different n are different and can be obtained using Satty table.
- f. Ealculate the CR = CI / RI. This value must be within 10 percent of the total index of 1, that is 0.1, to ensure that the comparison

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matrix [C] constructed by the decision maker is more consistent than the randomly populated matrix with values from 1 to 9 [3]. CR value under 0.1 is a green signal to proceed with the AHP process and criteria weights {W} for the attributes are accounted.

This process is repeated for each alternative with respect to each attribute. Size of the alternative comparison matrix is based on the number of alternatives. Since one alternative is compared with respect to each attribute, this becomes a lengthy process but is relatively simple. Each comparison matrix corresponding to each attribute gives a design alternative priority vector  $\{P_i\}$ . Design alternative priority vector with respect to each attribute gives a matrix called final rating matrix [FRating]. Matrix multiplication between [FRating] and criteria weight vector  $\{W\}$  is performed. This multiplication results in consolidated scores for each of the alternatives called material suitability index (MSI). The material with the highest MSI is the best material.

## Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a MCDM problem solving technique and was first developed by Hwang and Yoon [4]. This method is based on the concept that the best alternative to a problem from a set of available options will have the shortest Euclidean distance from the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). Euclidean distance between points p and q is defined as the length of the line segment connecting the points. In two dimensional measurements, this distance between the points is the absolute value of their numerical difference. However, if the number of points in the Euclidean space is n, then Equation 3 can be used to calculate the distance.

$$d = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2 + \dots + (p_n - q_n)^2}$$
  
Equation 3

The PIS is the hypothetical solution for which all attribute values correspond to the maximum attribute values comprising the satisfying solution, and NIS is the hypothetical solution for which all attribute values correspond to the minimum attribute values comprising the satisfying solution. TOPSIS thus gives a solution that is not only closest to the hypothetically best, but also farthest from the hypothetically worst [9].

The basic steps in TOPSIS that are taken for the selection of the best material from the set of short-listed materials are given as follows:

Step-1: Material selection attributes for the given engineering application are determined, and materials are short-listed on the basis of the identified attributes satisfying the requirements. Weighted decision matrix of size MxN, where M is the number of alternatives and N is the number of attributes, is created by using actual attribute values of each alternative with respective units. Each matrix element represented by  $m_{ij}$  gives the value of the j<sup>th</sup> attribute in original real values, that is, non-normalized form and units, for the i<sup>th</sup> alternative, or in short, incommensurable values.

Step-2: Euclidean distance from each of the elements in the columns to the origin is calculated using Equation 3. Normalized decision matrix  $R_{ij}$  is obtained using Equation 4. The term in the denominator is simply the Euclidean distance that has already been calculated.

$$R_{ij} = \frac{m_{ij}}{\left[\sum_{j=1}^{m} m_{ij}^{2}\right]^{1/2}}$$

Equation 4

Step-3: Next, weights of each attributes for the given application  $w_i$ , are determined using AHP.

In this assignment, either actual weighted values from AHP or corresponding weight in a given scale could be used. A weighted normalized matrix  $V_{ij}$ is obtained by multiplying  $w_j$  by  $R_{ij}$ . This allows to determine the PIS,  $V_j^+$  and NIS,  $V_j^-$  to the given problem. The PIS is a set of the best available options and NIS is a set of the worst available options in the weighted normalized matrix. These sets of options are represented by the expression given in Equations 5 and 6.

$$V_{j}^{+} = \{(\max(V_{ij}) \text{ if } j \in J); (\min(V_{ij}) \text{ if } j \in J')\}$$
$$V_{j}^{+} = \{V_{1}^{+}, V_{2}^{+}, V_{3}^{+}, \dots \dots \dots V_{m}^{+}\}$$

Equation 5

And, 
$$V_j^+ = \{(\max(V_{ij}) \text{ if } j \in J); (\min(V_{ij}) \text{ if } j \in J')\}$$
  
 $V_j^+ = \{V_1^-, V_2^-, V_3^+ - \dots \dots \dots V_m^-\}$ 

Equation 6

Where, J=(j=1,2,3,...,M) is associated with beneficial attributes

 $J'=(j=1,2,3,\ldots,M)$  is associated with non-beneficial attributes.

Referring to Equations 5 and 6, It may be added that PIS is a set of the smallest values of non-benefit attributes and the highest values of benefit attributes in the weighted normalized matrix for each alternative. In the case of NIS, that would be just the opposite.

Step-4: Once the positive and negative ideal solutions are obtained, positive separation measure  $(S_i^+)$  and negative separation measure  $(S_i^-)$  are calculated for each alternatives, once again using Euclidean distance as expressed in Equation 7.

$$S_{i}^{+} = \{ \Sigma_{j=1}^{M} (V_{ij} - V_{j}^{+})^{2} \}^{\frac{1}{2}}, i = 1, 2, 3, ..., N$$
$$S_{i}^{-} = \{ \Sigma_{j=1}^{M} (V_{ij} - V_{j}^{-})^{2} \}^{\frac{1}{2}}, i = 1, 2, 3, ..., N$$
Equation 7

Step-5: Finally, the relative closeness of a particular alternative to the ideal solution,  $P_i$  is calculated using the expression given in Equation 8.

$$P_{i} = \frac{S_{i}}{(S_{i}^{+} + S_{i})}$$

Equation 8

All the values of  $P_i$  are ranked in descending order: the alternative on the top is the best material and the value at the bottom is the worst material for the application.  $P_i$  value is sometimes also referred to as the performance score of alternative  $A_i$ .

#### Ashby's Approach

Ashby's charts are significant in materials selection for engineering design. Figure 1 [5] shows an example of Ashby's chart showing Young's modulus, E, plotted against density,  $\rho$ . It is visually clear that magnesium alloys are the lightest of the metal alloys shown but have the least stiffness, while titanium and steel alloys have the most stiffness but are much heavier. It could be very much appreciated from the plot alone that aluminum alloys could be the optimum metal alloy for a design that needs to be lighter and at the same time has a very good strength-to-weight ratio.

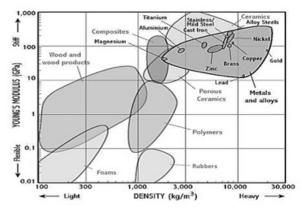


Figure 1: Ashby's chart - Young's modulus (E) plotted against density,  $\rho$  [5]

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While selecting materials, cost is one of the critical factors since companies are always looking to cut overall production cost without compensating other important factors. In this regard, combination of such plots involving all relevant material attributes satisfying the design requirements can very well predict the best material among the short-listed materials.

#### **Material Indices**

A material index is a combination of material properties which characterizes the performance of a material in a given application [5]. The design of a structural element is specified by three things: the functional requirements, the geometry, and the properties of the material of which it is made. The performance of the structural element is described by an expression of the form given in Equation 9.

$$p = f \begin{bmatrix} (Functional requirements, F), \\ (Geometric parameters, G), \\ (Materical Properties, M) \end{bmatrix}$$
$$p = f(F, G, M)$$

#### Equation 9

Where, p describes some aspect of the performance of the component: it's mass, or volume, or cost, or life for example; and f means a function of optimum design. Optimum design is the selection of the material and geometry which maximizes or minimizes p. Therefore, the above equation can be further written in the form given in Equation 10 [5].

$$p = f_1(F) f_2(G) f_3(M)$$

#### Equation 10

Where,  $f_1$ ,  $f_2$ ,  $f_3$  are separate functions which are simply multiplied together.

In an engineering design, a material property alone does not explicitly explain the performance

of a component. It is often a combination of two or even more that best describe the performance, hence allowing the design engineer to best select the material meeting the requirements [5]. Among material attributes that are considered for the design, a higher value of some of them is desired, and therefore such attributes are called benefit attributes. On the other hand, a smaller value of some of the attributes is desired, and therefore such attributes are called non-benefit attributes. For a design that requires a material with lighter weight and higher strength, a material with higher strength-toweight ratio, that is a material with lower density and higher Young's modulus is preferred. Since smaller value of density is desired, it is called a non-benefit attribute. Similarly, since a higher value of Young's modulus is desired, it is called a benefit attribute. Together both Young's modulus, E, and density, p, yield a material index for that particular material given as  $E/\rho$ . Any particular index for a given material is a constant number as given in Equation 11. Maximizing the value of this index maximizes stiffness at a minimum weight as an objective for the design.

For a particular material,

$$\frac{E}{\rho}$$
 = Constant (C)

Equation 11

Taking logs on both sides, Equation 11 can be written in the form of expression given in Equation 12.

$$\log(E) = \log(\rho) + \log(C)$$

#### Equation 12

This is an equation of a straight line of slope 1 on a plot of log(E) against log( $\rho$ ). Figure 2 [6] shows a plot of E against  $\rho$  in log-log scale describing the objective of stiffness at a minimum weight at a different level.

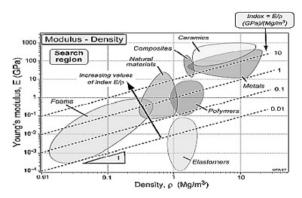


Figure 2: chart showing material index  $E/\rho$  describing the objective of stiffness at minimum weight [6]

A grid of lines corresponding to values of  $E/\rho$  from 0.1 to 10 in units of GPa/(mg.m<sup>3</sup>) are shown in the figure. It is now easier to read the subset of materials that maximize performances, meaning they have the highest values of  $E/\rho$  [5]. All the materials that lie on a line of constant  $E/\rho$  perform equally well as light, stiff components, those above the line perform better, and those below the line perform less. A material with the value of  $E/\rho = 10$ in these units gives a component with one tenth the weight for a given stiffness of a material with the value of  $E/\rho = 1$ .

### APPLICATION OF MATERIALS SELECTION METHODOLOGIES

After reviewing previous work carried out in the area of materials selection decision making as part of the literature review, several methodologies of materials selection are taken into consideration for the application. Some of the methodologies reviewed were: analytical hierarchical process (AHP), technique for order of preference by similarity to ideal solution (TOPSIS), compromised ranking method, and graph theory and matrix approach proposed by Rao [10]. Most of these methodologies have been briefly discussed in the literature review section of this article. Two of the methodologies, AHP and TOPSIS, are used to perform MCDM on the set of short-listed materials given in Table 2. New methodology using Ashby's approach is derived to rank materials to select the best materials and results are compared with those obtained from AHP and TOPSIS.

# Short-Listed Materials and Relevant Attributes

The objective in the design of lightweight aircraft metallic structures is to reduce weight, increase numerical values of all, if not most, mechanical properties, while cutting cost. Based on these basic requirements, material density (D), yield strength (YS), tensile strength (TS), Young's modulus (YM), fracture toughness in T-L (transverse-longitudinal) direction (FT), and cost (C) are considered as relevant attributes. A list of materials satisfying these requirements can be shortlisted. These materials that satisfy the requirements are short-listed and are given in Table 2 with their respective values with units of the attributes considered for the design. The ultimate goal is to find the best material among the short-listed materials using MCDM techniques and Ashby's approach. Among the materials short-listed, Al 7075-T651 and Al 2024-T4 among others are presently used by industries in lightweight aircraft metallic structures. Al 2024-T6 and Al 2024-T81 are short-listed as alternative materials to potentially replace the ones currently in use. The pair of magnesium alloys Mg AZ31B and Mg AZ61A are short-listed based on their high strength-to-weight ratio, competitive Young's modulus, and much lower density. Magnesium alloys are short-listed also because of the fact that there has been a long-going discussion regarding use of these alloys in the aircraft parts as part of the overall aircraft weight reduction agenda. It would be interesting to see where in the ranking these materials would stand and if in fact there is

any feasibility of these alloys to substitute the use of aluminum alloys that are short-listed for the design.

Table 2: table showing short-listed materials and relevant attributes with numerical values\*

	Relevant Attributes						
Alternatives	D	YS	TS	YM	FT	С	
AL 7075-T651	2.78	345	421	71	26.9	2.25	
AL 2024-T4	2.74	248	359	72	38	2.43	
AL 2024-T6	2.75	345	427	72	37	2.43	
AL 2024-T81	2.75	372	421	72	37	2.43	
Mg AZ31B	1.77	150	235	44	16	3.7	
Mg AZ61A	1.8	165	285	44	16	3.65	

D, density (g/cm<sup>2</sup>); YS, yield strength (MPa); TS, tensile strength before failure (MPa); YM, Young's modulus (GPa); FT, fracture toughness in T-L (transverse - longitudinal) direction in MPa $\sqrt{m}$ ; C, cost (\$/Kg)

Alternative materials and considered attributes

Table 3:	criteria	comparison	matrix	[C]
----------	----------	------------	--------	-----

given in this study are designated for a particular component in lightweight aircraft metallic structure that requires materials with high strength-to-weight ratio. Number of alternatives and attributes could slightly differ with varying component of any given lightweight aircraft metallic structure.

#### **Application of AHP Methodology**

AHP is used to select the best material for the design. The basic requirements are that the materials must be light weight and cost effective as non-benefit attributes. Unlike non-benefit attributes, materials must have high Young's modulus, high yield strength, high tensile strength, and high fracture strength as benefit attributes.

Table 2 displays the non-normalized numerical values with respective units of all the attributes for the short-listed materials. A pairwise comparison between one attribute to another is performed. Weights are assigned on the basis of degree of relative importance scale given in Table 1, and a criteria comparison matrix [C] is created as given in Table 3.

Materials selection factors	D	YS	TS	YM	FT	С
D	1.00	0.33	0.33	0.14	0.33	3.00
YS	3.00	1.00	3.00	0.20	0.33	7.00
TS	3.00	0.33	1.00	0.14	0.20	5.00
YM	7.00	5.00	7.00	1.00	3.00	9.00
FT	3.00	3.00	5.00	0.33	1.00	5.00
С	0.33	0.14	0.20	0.11	0.20	1.00

\*ASM International, Alloy Center Database, (mio.asminternational.org/ac/index. aspx?profileKey=grantami\_ac\_alloyfinder)

An attribute compared to itself is always one. Yield strength compared to density is given slightly more importance. Even though density is an important attribute in the design, yield strength of the material cannot be compromised for the lighter weight due to components' reliability and other safety reasons. A similar argument applies to the cost. No matter how important it is to reduce production cost, it can never be compromised with mechanical attributes whose higher values are always desired. It is sometimes harder to perform pairwise comparison among the mechanical attributes of the materials. In such situations, one has to decide whether the components require a better fracture toughness or tensile strength and so forth.

Matrix [C] is normalized by dividing each element in the matrix with its respective column total and a new matrix is created called normalized weighted matrix [Norm C] and is given in Table 4. The average of each rows gives the criteria weight vector {W} for each attribute in the design. According to {W}, Young's modulus is the most important attribute. Fracture toughness, Yield strength, tensile strength, and density follow Young's modulus in the order, while cost turns out to be the least important.

Materials selection factors	{W}	{W <sub>s</sub> }	{Cons}
D	0.0582	0.3600	6.1812
YS	0.1432	0.9602	6.7040
TS	0.0911	0.5675	6.2319
YM	0.4586	3.1422	6.8523
FT	0.2196	1.5770	7.1798
С	0.0293	0.1811	6.1894

*Table 5: summary of {W}, {W<sub>s</sub>}, and {Cons}* 

Criteria weight vector {W} describes the individual weights of each attribute affecting the design. A consistency check is performed to ensure the consistency in pairwise comparison in the matrix [C]. This process has been explained in the previous chapter and results are given in Table 5.

Weighted sum vector is calculated as  $\{W_s\} = [C] \{W\}$ . To do this, multiplication between the criteria comparison matrix [C] and criteria weight vector  $\{W\}$  is performed. This is simply the sum of

Materials selection factors	D	YS	TS	YM	FT	С	Criteria weight vector {W}
D	0.0577	0.0337	0.0200	0.0729	0.0652	0.1000	0.0582
YS	0.1731	0.1020	0.1815	0.1042	0.0652	0.2333	0.1432
TS	0.1731	0.0337	0.0605	0.0729	0.0395	0.1667	0.0911
YM	0.4039	0.5102	0.4235	0.5208	0.5929	0.3000	0.4586
FT	0.1731	0.3061	0.3025	0.1719	0.1976	0.1667	0.2196
С	0.0190	0.0143	0.0121	0.0573	0.0395	0.0333	0.0293

Table 4: normalized weighted matrix [Norm C] and Criteria weight vector  $\{W\}$ 

the product of each row in [C] and column in  $\{W\}$ . This provides the weight sum vector  $\{W_s\}$ . Consistency vector  $\{Cons\}$  is determined by multiplying  $\{W_s\}$  with the reciprocal of  $\{W\}$ .

Average value of the consistency vector {Cons} is calculated to be 6.53 and is called the Eigen value of the matrix,  $\lambda$ . Consistency Index (CI) is calculated using Equation 2 and is 0.106793. Random Index (RI) value of 1.25 for n = 6 is obtained from Satty table. Finally, CR = CI/RI is calculated to be 0.0854, which is less than 0.1, meaning the consistency is greater than 90 percent and is acceptable for the process. This indicates the pairwise comparison weights assigned are consistent, and the process may continue. Once CR in the matrix is checked for

consistency, the criteria weights vector  $\{W\}$  for the attributes is finalized.

This process is entirely repeated for a pairwise comparison among alternative materials with respect to each attribute. Consistency check is performed for each comparison to ensure the validity of the decision maker's decision in assigning weights to one alternative to another. For demonstration propose, pairwise comparison among alternatives with respect to density is performed as given in Table 6.

Using the same procedure as the one used to calculate  $\{W\}$  in attribute pairwise comparison, the priority vector  $\{P_i\}$  is calculated. Vector  $\{P_i\}$  provides percentage weight of each short-listed materials with respect to individual attribute. Table

Table 6: pairwise comparison among alternatives with respect to density, p

Alternative materials	Al 7078-T651	Al 2024-T4	Al 2024-T6	Al 2024-T81	Mg AZ31B	Mg AZ61A
Al 7075-T651	1.00	0.20	0.33	0.33	0.11	0.14
Al 2024-T4	5.00	1.00	3.00	3.00	0.20	0.33
AI 2024-T6	3.00	0.33	1.00	1.00	0.14	0.20
Al 2024-T81	3.00	0.33	1.00	1.00	0.14	0.20
Mg AZ31B	9.00	5.00	7.00	7.00	1.00	3.00
Mg AZ61A	7.00	3.00	5.00	5.00	0.33	1.00

Table 7: normalized weighted matrix [Norm C] and priority vector  $\{P_i\}$  with respect to density

Alternative materials	Al 7078- 651	Al 2024- T4	Al 2024- T6	Al 2024- T81	MG AZ31B	Mg AZ61A	The priority vector {Pi}
Al 7075-T651	0.04	0.02	0.02	0.02	0.06	0.03	0.0300
Al 2024-T4	0.18	0.10	0.17	0.17	0.10	0.07	0.1330
AI 2024-T6	0.11	0.03	0.06	0.06	0.07	0.04	0.0617
Al 2024-T81	0.11	0.03	0.06	0.06	0.07	0.04	0.0617
Mg AZ31B	0.32	0.51	0.40	0.40	0.52	0.62	0.4622
Mg AZ61A	0.25	0.30	0.29	0.29	0.17	0.21	0.2514

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7 shows the priority vector of alternative pairwise comparison with respect to density.

Pairwise comparison among all the alternatives with respect to each attribute is completed. CR is calculated in each of these comparisons and confirmed that CRs for each pairwise comparison is less than 0.1. Priority vector of all the comparison is combined to obtain a Final Rating Matrix [FRating] and is given in Table 8 along with {W}.

Finally, the matrix multiplication between [FRating] and {W} is performed yielding the

Table 9: table showing calculated MSI values and corresponding material ranking

Materials	MSI	Ranking
Al 7075-T651	0.1373	4
AI 2024-T4	0.2478	2
Al 2024-T6	0.2427	3
Al 2024-T81	0.2520	1
Mg AZ31B	0.0637	5
Mg AZ61A	0.0565	6

material suitability index (MSI). Material with the highest MSI is the best material for the design. Summary of this calculation and ranking of each alternative is given in Table 9.

Using this methodology, Al 2024-T81 is the best material. Both magnesium alloys are not the suitable materials for the design despite their light weight.

#### **Application of TOPSIS Methodology**

A decision matrix is created using actual material attribute values given in Table 2 that are incommensurable. Euclidean distance from each of the attribute values in the column to the origin is calculated using Equation 3. Decision matrix is normalized by dividing each element  $m_{ij}$  in the column with their respective Euclidean distances as given by Equation 4 and a new matrix given in Table 10 is created. This matrix is called normalized decision matrix  $R_{ij}$ .

Next step is to weigh on the individual attributes. To carry on this task, each attribute is given certain weight  $w_j$  based on their importance satisfying the design requirements. In order to be consistent with weighing on attributes, AHP is exercised. Criteria weight vector  $\{W\}$  that was calculated previously in the AHP is used for this purpose. It is critical to know that weights of attributes could arbitrarily be assigned

Table 8: final rating matrix [FRating] with criteria weight vector {W	Table 8: fir	nal rating matrix	[FRating] w	vith criteria	weight vector {	W
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		Material selection attributes						
Short-listed materials	D	YS	TS	YM	FT	С	weight vector {W}	
Al 7075-T651	0.0302	0.2152	0.2003	0.1364	0.0935	0.1148	0.0582	
AI 2024-T4	0.1330	0.0846	0.0981	0.2578	0.4237	0.2655	0.1432	
AI 2024-T6	0.0619	0.2152	0.4228	0.2578	0.1992	0.2655	0.0911	
Al 2024-T81	0.0619	0.4216	0.2003	0.2578	0.1992	0.2655	0.4586	
Mg AZ31B	0.4615	0.0242	0.0276	0.0451	0.0422	0.0310	0.2196	
Mg AZ61A	0.2515	0.0392	0.0508	0.0451	0.0422	0.0578	0.0293	

within a given scale and could very well change from one decision maker to another.

Multiplication of  $R_{ij}$  in the column with their respective wj gives the weighted normalized decision matrix. This matrix is presented in Table 11.

PIS and NIS are obtained from the table using Equations 5 and 6. PIS is a set of highest values of benefit attributes and lowest values of non-benefit attributes from each column. Similarly, NIS is a set of lowest values of benefit attributes and highest values of non-benefit attributes. This gives;  $PIS = \{0.0164,$  0.0764, 0.0436, 0.2114, 0.1137, 0.0094} and NIS =  $\{0.0257, 0.0308, 0.0240, 0.1292, 0.0479, 0.0155\}$ . Using Equation 6, both positive and negative separation measures,  $S_i^+$  and  $S_i^-$ , are calculated. A summary of separation measures, their sum, and calculation of relative closeness to the positive ideal solution is given in Table 12. Rankings based on the relative closeness of alternative materials to the ideal solution are also included in the table.

According to this methodology, Al 2024-T81 is the best material which agrees with that from the

Alternatives	Relevant Attributes						
Alternatives	D	YS	TS	YM	FT	С	
Al 7075-T651	0.4587	0.4940	0.4701	0.4539	0.3655	0.3189	
Al 2024-T4	0.4521	0.3551	0.4008	0.4603	0.5163	0.3444	
Al 2024-T6	0.4537	0.4940	0.4768	0.4603	0.5027	0.3444	
AI 2024-T81	0.4537	0.5327	0.4701	0.4603	0.5027	0.3444	
Mg AZ31B	0.2920	0.2148	0.2624	0.2813	0.2174	0.5244	
Mg AZ61A	0.2970	0.2363	0.3182	0.2813	0.2174	0.5173	

Table 11: weighted normalized decision matrix

Alternatives	Relevant Attributes						
Alternatives	D	YS	TS	YM	FT	С	
Al 7075-T651	0.0257	0.0709	0.0430	0.2085	0.0805	0.0094	
Al 2024-T4	0.0253	0.0510	0.0367	0.2114	0.1137	0.0102	
Al 2024-T6	0.0254	0.0709	0.0436	0.2114	0.1107	0.0102	
Al 2024-T81	0.0254	0.0764	0.0430	0.2114	0.1107	0.0102	
Mg AZ31B	0.0164	0.0308	0.0240	0.1292	0.0479	0.0155	
Mg AZ61A	0.0166	0.0339	0.0291	0.1292	0.0479	0.0153	

AHP. Even though, rest of the rankings do not quite agree with that from AHP. While rankings from one method to another need not be the same, most of the rankings are expected to agree, especially they on the best material and that was exactly the case here.

#### **Application of Ashby's Approach**

Under Ashby's approach, which as has been discussed, involves the significance of benefit and nonbenefit attributes in the design, it is important to recognize the differences between attributes while determining the material indices. The objective is always to maximize the value of benefit attribute and minimize that of non-benefit attribute. Among six attributes considered, density and cost are identified as non-benefit attributes and the rest of the attributes are identified as benefit attributes. Based on the classification of attributes in terms of what needs to be minimized or maximized, the following material indices are identified and are to be maximized. Maximum value of each of the indices listed below will perform at an optimum level by a component in a given lightweight aircraft metallic structures:

- 1. Young's modulus versus density  $(E/\rho)$
- 2. Young's modulus versus cost (E/C)
- 3. Yield strength versus density ( $\sigma y / \rho$ )
- 4. Yield strength versus cost ( $\sigma y / C$ )

- 5. Tensile strength versus density ( $\sigma F / \rho$ )
- 6. Tensile strength versus cost ( $\sigma F/C$ )
- 7. Fracture toughness versus density (K1C  $/\rho$ )
- 8. Fracture toughness versus cost (K1C /C)

If Ashby's charts are created for each of the above indices by plotting one attribute versus the other, materials that perform equally well with respect to each of the indices could be located. For each index plot, precisely focusing in the region where aluminum and magnesium alloys are located, and if indeed shortlisted material in this study are found in the same location, it would be fair to say that the ranking based on the performance of individual material index values gives the best material for the design. In addition, as described previously in reference to Figure 2, a grid of lines could be drawn parallel to each of the straight lines produced by individual indices in a log-log scale and an attempt could be made to locate magnesium and aluminum alloys in the region at close proximity to the grid lines. This would be another attempt to locate material matching the short-listed materials that are used in this study. Obviously, without using a material selection software that incorporates Ashby's charts, this task would be very difficult to execute. Using the individual attribute values given in Table 2, values of all of the above the indices are calculated and are given in Table 13.

Alternatives	$S_i^+$	S <sub>i</sub> -	$S_{i}^{+}+S_{i}^{-}$	P <sub>i</sub>	Ranking
Al 7075-T651	0.0351	0.0967	0.1318	0.7338	4
Al 2024-T4	0.0279	0.1081	0.1359	0.7950	2
AI 2024-T6	0.0111	0.1128	0.1238	0.9107	3
Al 2024-T81	0.0096	0.1148	0.1244	0.9228	1
Mg AZ31B	0.1166	0.0093	0.1259	0.0742	5
Mg AZ61A	0.1146	0.0109	0.1255	0.0865	6

<i>Table 12:</i>	calculated	separation	measures	and Pi values

Short-listed			Materia	indices				
Materials	E/ρ	E/ C	σ,/ρ	σ <sub>y</sub> /C	$\sigma_{_F}/ ho$	$\sigma_{_{\rm F}}/{\rm C}$	K <sub>1C</sub> /ρ	K <sub>1C</sub> /C
Al 7075-T651	25.54	31.56	124.10	153.33	151.44	187.11	9.68	11.96
AI 2024-T4	26.28	29.63	90.51	102.06	131.02	147.74	13.87	15.64
Al 2024-T6	26.18	29.63	125.45	141.98	155.27	175.72	13.45	15.23
Al 2024-T81	26.18	29.63	135.27	153.09	153.09	173.25	13.45	15.23
Mg AZ31B	24.86	11.89	84.75	40.54	132.77	63.51	9.04	4.32
Mg AZ61A	24.44	12.05	91.67	45.21	158.33	78.08	8.89	4.38

Table 13: eight different material indices and their values for each alternative

Table 13 shows that each of the material indices is a different value for each material. Since the maximum value of each of the index is desired, the material with the highest index value in each category is the best material. For example, while maximizing  $E/\rho$ , Al 2024-T4 would be the best material, but maximizing E/C would make Al 7075-T651 the best material. If all the materials are ranked based on individual index values, different materials would perform differently. In order to identify a single best material for the design with respect to all the indices, their individual ranking could be averaged. Since the best material receives a ranking of one, the material with the least average ranking value could be identified as the best material. This approach has been applied to the short-listed materials in this study and results are summarized below in Table 14.

From the table, it is apparent that different materials rank differently with respect to individual material index. For example, Mg AZ61A ranks as the best material with respect to tensile strength versus density. That means if a design requires high tensile strength and low density material, Mg

Table 14: Individual ranking of materials based on eight different indices

Short-listed				Material	indices			
Materials	E/ρ	E/ C	σ͵/ρ	σ <sub>y</sub> /C	σ <sub>F</sub> /ρ	$\sigma_{_{\rm F}}/{\rm C}$	K <sub>1C</sub> /ρ	K <sub>1C</sub> /C
Al 7075-T651	3	1	1	1	4	1	4	3
Al 2024-T4	1	2	5	4	6	4	1	1
AI 2024-T6	2	2	2	3	2	2	2	2
Al 2024-T81	2	2	1	2	3	3	2	2
Mg AZ31B	4	4	6	6	5	6	5	5
Mg AZ61A	5	3	4	5	1	5	6	4

AZ61A would be the best material given no other constraints remain active, which is not very likely in any design. Rankings of materials with respect to each material index is averaged. Material with ranking one is the best material and ranking 6 is the worst material. Therefore, the material that has the least average ranking number is the best material. Summary of average ranking and ultimate material ranking using this approach is presented in Table 15.

*Table 15: Average ranking of materials and ultimate ranking of materials* 

Short-listed Materials	Average of ranking	Ultimate ranking
Al 7075-T651	2.500	3
Al 2024-T4	3.000	4
Al 2024-T6	2.125	2
Al 2024-T81	2.125	1
Mg AZ31B	5.125	6
Mg AZ61A	4.125	5

According to this approach, Al 2024-T81 is the best material which perfectly agrees with the results obtained using TOPSIS as well as AHP. It should also be mentioned that ranking using this approach, both Al 2024-T6 and Al 2024-T81 rank similarly. In either case, AL 2024-T81 can very well be selected

as the best material for the design. Overall ranking of materials using this approach significantly agree with that from TOPSIS. Since, TOPSIS is a reliable and promising MCDM technique that is widely used in materials selection and results from Ashby's approach are very similar to TOPSIS, it can be said that this new approach of material selection using Ashby's approach is indeed a reliable technique in materials selection for lightweight aircraft metallic structures. This technique is very simple and easy to understand. Having said that, there must be a clear understanding of all the relevant material indices in terms what is to be maximized as well minimized.

## **RESULTS AND DISCUSSION**

Ranking results using AHP and TOPSIS along with new methodology in material selection using Ashby's approach are summarized and presented in Figure 3 for visual interpretation. It is easier to read off the ranking from the individual plots given in the figure. Al 2024-T81 has the best ranking of all the short-listed materials while both magnesium alloys rank the last.

In an approach to combine the individual ranking results of materials using three different methodologies, a plot given in Figure 4 is generated. It is even easier from this combined plot to visualize the comparison and determine that the best material is Al 2024-T81 for all the methodologies. From the plot it is also clear that the last ranking materials are the ones from magnesium alloy group.

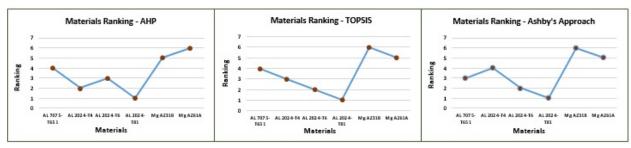


Figure 3: Individual ranking of materials using three different methodologies

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'Figure 4: Summary of ranking results incorporating all three rankings

The intuition is that regardless of the methodologies used to select the best materials for the design, the outcome must be the same. However, comparing the results obtained using two different existing methodologies and new methodology using Ashby's approach produce results with certain degree of variances. In the real world, these variances are well expected. The most important fact of the three measures of ranking in this study is that they all agree on the best material as well as last two alternatives being magnesium alloys, which are shown not to be suitable for the design. Among these three methodologies, each one has both pros and cons. Only attributes that have actual quantitative values were considered in this study. If a design requirement for a certain part in lightweight aircraft metallic structures has to consider attribute that do not have quantitative values such as machining rating or corrosion scale, a qualitative measure has to be defined. AHP as well as TOPSIS can efficiently define such qualitative measures in pairwise comparison using fuzzy numbers conversion within a given scale. On the other hand, new methodology under Ashby's approach fails to accommodate any qualitative measures in the process. When number of attributes or the alternatives increase significantly, AHP becomes highly complicated to keep track of pairwise comparison while TOPSIS and generalized Ashby's approach of materials selection can handle any number of attributes and alternatives without any difficulty. Despite the weakness in addressing qualitative measures, the advantageous characteristics of the new generalized methodology of materials selection using Ashby's approach proposed in this paper are summarized below:

- 1. The new methodology can handle any number of quantitative attributes and alternatives and offers simple logical approach in materials selection for any component in lightweight aircraft metallic structures.
- 2. The methodology always involves the implication of material indices identifying non-benefit and benefit attributes and determines whether the index value should be maximized or minimized.
- 3. The methodology also determines the best materials based on individual index values and eventually the best material considering an aggregate of all the material indices values using their average.

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4. The best material has the least average ranking and materials not suitable for the design have the higher average ranking values.

#### CONCLUSIONS

Understanding of KBS and its implementation in materials selection for lightweight aircraft metallic structures using various existing MCDM methodologies remained the focus in this study. Much literature in the area of materials selection and decision making in engineering design was reviewed. Material attributes as data and the information in the data about the material collectively known as KBS was essential in the study. It was critical to identify the most relevant attributes to satisfy the design requirements for any lightweight aircraft metallic structures. Short-listing of materials was made based on two materials from aluminum alloy group known to have been used by industries in the design of components for lightweight aircraft metallic structures and other four with attributes very close to the reference materials. Among various multi-criteria decision making methodologies, AHP and TOPSIS were used to rank short-listed materials that perform the best. Ashby's approach was generalized to develop a new methodology in materials selection by determining all relevant material indices and ranking materials based on individual material indices values. The newly developed methodology in material selection is simple and can incorporate unrestricted number of alternatives and attributes. Results from this methodology very closely agree with that from TOPSIS but not quite closely with that from AHP. However, the best material for the lightweight aircraft metallic structure is deemed to be the same from all the methodologies.

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